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About the Notion of Truth in Quantum Mechanics

Roland Omnès[1]

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The meaning of truth in quantum mechanics is considered in order to respond to some objections raised by B. d'Espagnat against a logical interpretation of quantum mechanics recently proposed by the author. A complete answer is given. It is shown that not only can factual data be said to be true, but also some of their logical consequences, so that the definition of truth given by Heisenberg is both extended and refined. Some nontrue but "reliable" propositions may also be used, but they are somewhat arbitrary because of the complementarity principle. For instance, the propositions expressing wave packet reduction can be either true or reliable, according to the case under study. Separability is also discussed: as far as the true properties of an individual system are concerned, quantum mechanics is separable.

KEY WORDS: Quantum mechanics; foundations; truth; separability.

In a recent paper, "Are there realistically interpretable quantum theories?,"[1] d'Espagnat raised some objections against an interpretation of quantum mechanics recently proposed by the author,[2,3] to be called here the logical interpretation. His main criticism was directed against the notion of truth in this theory and his main concern was to maintain the nonseparability of quantum mechanics. Although this might look like a philosophical question to a casual reader, it is really a matter of prime importance to physics, since, as will be shown, it involves among other questions the exact status of wave packet reduction. Another feature gives some further weight to these questions: It has become clear that the above theory, although initially built up independently of the Copenhagen interpretation, provides at the end a consistency proof of it. However, it stands upon fewer and clearer axioms, allowing one to derive completely the inter-

[1] Laboratoire de Physique Théorique et Hautes Energies (Laboratoire associé au CNRS), B.210 Université de Paris-Sud, F-91405 Orsay, France.

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pretation of the theory in a deductive way. Because everything can be proved, some Copenhagen assumptions must be corrected as found to be too strong to be completely general, so that their limits are delineated. Nevertheless, this is still essentially the Copenhagen interpretation, rediscovered in a different way. Accordingly, d'Espagnat's objections and questions have a much wider interest than if they had been directed against simply another "new" theory. They are basic questions concerning in fact the whole setup of quantum mechanics and answering them goes a long way toward an explicitly objective theory.

As far as my own work is concerned, there are two main points in d'Espagnat's paper. The first one is to decide whether or not the theory contradicts nonseparability and in particular Bell's theorem.[5,6] The second one has to do with the "realistic" character of the theory and more specifically what is the meaning of truth in it. I had cautiously avoided using the word "true" in my papers up to now, because I was aware of the difficulty of these problems and wanted time to solve them. This is why the word "reliable" was used everywhere instead of "true," with a meaning to be recalled and made still more precise shortly. To answer d'Espagnat's criticisms, I shall give here the results of an investigation of these questions that has been made in the meantime.

1. OUTLOOK

Since these are not easy questions, it may be useful to make clear what is really at stake. Neither Griffiths nor myself wrote anything about the separability of quantum mechanics, which was d'Espagnat's main concern. He noticed, however, that one might get an "impression" that separability was in some way holding in the logical interpretation and if so, this would have to be made clear. At that time, d'Espagnat did not believe that separability could be obtained, as his paper makes clear, because that would question some far-reaching reasonings concerning realism relying upon nonseparability as one of their arguments.[4,5] He certainly could be justified when trying to preserve this kind of result, even if they are of a philosophical nature. So, the problem of realism lies partly behind the discussion, although I shall avoid it.

The question of truth is essential here because separability, as explained in the following, refers to some properties of an isolated system that might be influenced by what occurs to another system. Whether these properties are true, or more strongly said, real, is the root of the question.

The organization of the present discussion is the following: It is necessary for clarity to recall first very briefly the general background. One will find in Section 2 the basic assumptions of the logical interpretation; in

Section 3 the general theory of classical facts in this theory; in Section 4 what is a measurement in this framework; and in Section 5 how actual facts enter.

Actual, empirical facts should be held as true. In Section 6, it is shown that some other properties of a quantum system can be also consistently said to be true, whereas other so-called reliable properties may be put forward, but are somewhat arbitrary. This is applied in Section 7 to the Einstein–Podolsky–Rosen–Bohm situation to show that in that case conventional wave packet reduction and EPR "elements of reality" are reliable, but not true, i.e., they are arbitrary, although not self-contradictory. This is used in Section 8 to show that quantum mechanics is separable as far as true properties are concerned. Section 9 says a few words about Bell's theorem, which was at the center of d'Espagnat's argument, to explain why it has essentially little to do with the present theory.

2. THE FRAMEWORK

In the logical interpretation, one assumes that all the meaningful physical properties of a physical system can be expressed in terms of Griffiths histories.(7) A Griffiths history describes some properties of an isolated quantum system at different times $t_1, t_2,..., t_n$. It may be conveniently thought of as a class of Feynman histories for which the range of values of some observables are restricted at these times. More precisely, the state of the system is given by a density operator ρ at a time zero. The reference times are ordered: $0 < t_1 < t_2 < \cdots < t_n$. For each time, the stated property refers to an observable, say $A_1, A_2,..., A_n$. The spectrum σ_k of each observable A_k is divided into several disjoint sets D_k^α. All these conventions being made once and for all, a history is a set of n von Neumann predicates(8) saying that "the value of the observable A_k is in the set D_k^α at time t_k" for each $k = 1,..., n$. As usual, each predicate can be associated with a Heisenberg time-dependent projector $E_k^\alpha(t_k)$. A probability is assigned to such a history, namely

$$p = \mathrm{Tr}\{E_n^\nu(t_n) \cdots E_2^\beta(t_2 E_1^\alpha(t_1)\, \rho E_1^\alpha(t_1) \cdots E_n^\nu(t_n)\} \tag{1}$$

a formula arising naturally from a sum upon Feynman histories.(9)

The case $n = 2$ being simplest, it will be taken as an example. The set $\Xi = \sigma_1 \times \sigma_2$ can be divided into elementary boxes $D_1^\alpha \times D_2^\beta$, each box corresponding to a history. The different histories can be treated as elementary events in probability calculus. Any set of such boxes is considered as a proposition and it turns out that all these propositions are enough to describe the whole events of physics. The basic logical operations "and, or" are associated with the intersection or the union of the corresponding subsets of Ξ, negation with the complement of a set. A proposition a is said to imply another proposition b ($a \Rightarrow b$) when their conditional probability satisfies the condition

$$p(b \mid a) = 1 \tag{2}$$

One says that a implies b with error ε (ε a small positive number) when

$$p(b \mid a) > 1 - \varepsilon \tag{3}$$

Furthermore, a is said to be logically equivalent to b if $a \Rightarrow b$ and $b \Rightarrow a$.

When the initial state, the reference times, the associated observables, and the cutting of their spectra are given, one has defined a field of propositions together with the basic rules of logic on that field. This is called a logic.

However, it turns out that the probability given by Eq. (1) does not always satisfy the additivity rule of probability calculus. It only does when the addition of probabilities is consistent with the addition of probability amplitudes. This strong restriction leads to a set of consistency conditions first established by Griffiths,[7] which read, in the case $n = 2$,

$$\operatorname{Re} \operatorname{Tr}\{E_1^\alpha(t_1)\, \rho E_1^\gamma(t_1)\, E_2^\beta(t_2)\} = 0 \qquad \text{when} \quad \alpha \neq \gamma \tag{4}$$

One can also accept that these conditions are satisfied only up to a small relative error. It turns out that they also imply that the logical rules satisfy all the formal axioms of logic. This is why a logic satisfying them can be called a consistent logic.

The whole interpretation of the theory is based upon a unique and universal logical rule according to which any description of the properties of an isolated system should consist of propositions belonging to a consistent logic and any reasoning about these properties should consist of a chain of valid implications.

This is the framework within which the discussion of the meaning of truth will take place. Despite the construction of a neat logical setup and the ensuing reliability of ordinary formal logic, the question is nontrivial because there are so many possible consistent logics which have nothing much in common. This multiplicity is in fact the expression of the complementarity principle. There is a useful noncontradiction theorem according to which, if two consistent logics L and L' both contain the same two propositions a and b in their respective fields and if $a \Rightarrow b$ in L, then $a \Rightarrow b$ in L'. It means that there cannot be any logical conflict, i.e., no paradox in quantum mechanics if one is careful enough! The main point of the discussion in the present paper is to make sure that one has been careful enough.

3. THE THEORY OF FACTS

The most difficult technical part of the theory is to build up a consistent semiclassical physics in order to encompass the "classical" properties of a macroscopic system within the framework. It may be called a theory of potential facts (for a logician, a potential fact is a property of the system sharing most properties of ordinary empirical facts, except that it remains a hypothetical proposition, i.e., a mere sentence whose construction is allowed by the theory; in that sense, it belongs to what a logician would call an object language).

It will be convenient to express the main features of a potential fact by stating that: (i) It describes a property of a macroscopic system that can be expressed equivalently in classical physics and in quantum mechanics, except for a small error in probability. (ii) Classical determinism relates it to similar properties at previous or later times; this kind of determinism also holds in quantum mechanics, where it is a logical implication involving a small, computable error in probability. (iii) Two different potential facts are neatly separated by decoherance.[12–15] (iv) There is no possible measurement allowing to one find out some quantum coherence between two different facts. In a simpler way, one can say that potential facts as described by quantum mechanics do exist, they are deterministic, neat, and unescapable.

I refer to recent papers[3,10] where a proof of the two first points is given, to mention here only the most salient features. One considers a macroscopic object in a state allowing one to define some collective coordinates together with microscopic coordinates which parametrize a so-called internal environment. Let Q be the collective coordinates and N be their number, the canonically conjugate observables being P. A typical classical proposition would state the values of Q and P with some prescribed errors, i.e., it would place (Q, P) in some cell C in phase space which is big in terms of Planck's constant. One can associate a quantum predicate to such a classical statement by assigning it a projector in Hilbert space. The rank of this projector (i.e., the dimensionality of the corresponding subspace of the Hilbert space) is essentially the number of semiclassical states one can pile up within the cell C. There is a slight freedom in the definition of this projector, two such allowed projectors F and F' differing only in such a way that

$$\mathrm{Tr}\,|F - F'|/\mathrm{Tr}\,F = O(\varepsilon) \tag{5}$$

ε being a small quantity depending only upon the cell (note the occurrence of a trace norm). For instance, when C is a box having a length L along the Q-directions and Π along the P-directions, then ε is equal to $(\hbar/L\Pi)^{1/2}$ whatever N. The existence of this family of projectors is what is meant by condition (i). There are some other geometrical conditions to be met by the cell for this result to be true. Let us call them regularity conditions and refer to previous papers[3,10] for their explicit statement.

The origin of determinism is the following: Given a collective Hamiltonian H_c, one can derive from it a Hamilton function $h(Q, P)$ by using the Wigner–Weyl recipe and accordingly define Hamilton equations and classical motion in phase space. A cell C such as the previous one becomes another cell C' under classical motion during a time interval t. The dynamics is said to be regular for the cell C and the time t when essentially both C and C' are regular. This is a rather strong condition upon the function $h(Q, P)$ and therefore upon the collective Hamiltonian. Since C and C' are both regular, one can associate two projectors F and F' with them. It can be proved that one has

$$\mathrm{Tr}\,|F' - U(t)\,FU^{-1}(t)|/\mathrm{Tr}\,F = O(\varepsilon(t)) \tag{6}$$

where $\varepsilon(t)$ is a computable small quantity and $U(t)$ is the evolution operator $\exp(-iH_c t/\hbar)$. Equation (6) means that classical dynamics and quantum mechanics essentially agree for regular systems and regular cells. From this one can prove that classical determinism holds as a reciprocal quantum implication between the predicates expressing the deterministic relation between the two classical situations, these implications taking place within an approximately consistent logic. This is what is meant by condition (ii), so that determinism is recovered within quantum mechanics as far as logic is concerned. However, it only holds under precise and computable conditions with a known error.

Among the nonregular systems left aside by the proof of these results, one can mention the case of chaotic systems for which the time during which classical logic holds is finite and even rather short, so that one can only show a statistical agreement between the quantum description and the classical one.[10] It can also happen that some macroscopic systems are not prepared in an initial state allowing a classical description, as may be the case for a SQUID device.[11]

According to condition (iii), facts are also neatly separated, i.e., there is no effective quantum superposition of classically different states. As well known, this result follows from the decoherence effect[12–15]: The density operator of a macroscopic object, when restricted to describe only the collective observables by performing a partial trace upon its environment, becomes very rapidly diagonal, even if it represents initially a linear superposition of different macroscopic states. Again, there are exceptions to this behavior when decoherence does not occur or becomes very slow, as can happen in nondissipative systems such as a superconductor or a superfluid.[11,14] These cases show, by the way, that one cannot be any more completely satisfied by the older version of the Copenhagen interpretation, since it makes no room for such exceptions which have been seen experimentally.

Finally, facts are unavoidable and this is condition (iv). One might conceive the idea of beating decoherence by measuring very sharply some microscopic observable so as to exhibit experimentally a superposition of states that remained hidden within the complete density operator, although washed down by decoherence in the reduced density operator. By using the previous results together with the analysis of decoherence made by Caldeira and Leggett,[14] it can be shown that the observables whose measurement could contradict decoherence are rather restricted. They consist in measuring the occurrence of the internal environment in some precisely defined quantum eigenstate of energy. Since the theory of measurements one can build up by using the previously mentioned theorems[2] gives explicitly all errors, one can prove the following unpublished result: In order to beat decoherence when measuring a system having N degrees of freedom, one must use a measuring device involving at least N' degrees of freedom, where

$$N' > K \exp(K' N^{2/3}) \tag{7}$$

K and K' are constants depending upon the matter from which the second apparatus is made (ordinary metal, neutron star stuff, and so on). For ordinary metals, K is large and K' of the order of one. For N large enough, the measuring apparatus would be so big as to be unable to work as a whole just because of the time a signal would take to cross it, so that coherence can never be measured except for systems having just a few degrees of freedom or no dissipation. Therefore facts are unescapable.

4. MEASUREMENT THEORY

The discussion of d'Espagnat's objections also bears upon the consequences one may draw from a measurement. The main results of the theory on this subject must therefore also be recalled.[2] Let for instance, Q be some atom, and A an observable belonging to it (or a set of commuting observables), e.g., a spin component. There is no real loss in generality by assuming A to have only discrete eigenvalues. Let M be a measuring apparatus. It is assumed to possess a collective observable B whose value before and after measurement can be treated as a (potentiel) fact as defined previously. The initial value of B is zero (neutral position of the measuring device) and its final value is b_n when the initial state of Q is an eigenstate of A with eigenvalue a_n. These conditions can be stated precisely as semidiagonal properties of the quantum evolution operator for the system $Q+M$ during the time interval when Q and M are interacting. Let us denote by t the beginning of the measurement and by t' the end of it. The potential fact expressing that "the value of B is b_n at the end of the measurement" will be called the *data* D. The proposition stating that "the value of A is a_n at the beginning of the measurement" is called the *result* R of the measurement. For a measurement of type I where an initial value of A is unchanged by the interaction, one can also introduce the statement "the value of A is a_n at the end of the measurement" to be called the *reduction predicate* (denoted by Red) by analogy with wave packet reduction.

Several significant results were already obtained in the case where the eigenvalues of A are nondegenerate.[2] Although no calculation will be given explicitly in the present paper, it should be borne in mind that all of what will be said is based upon the universal logical rule of interpretation, i.e., it always relies upon a check of consistency conditions and the calculation of conditional probabilities. Despite its obvious interest, no discussion of wave packet reduction as such will be given here, since wave packet reduction can always be avoided. However, the reduction predicate can be used to express clearly an interesting property of the system and it will play an important role in the following.

When the eigenvalues of A are nondegenerate, it has been proved that there exists a consistent logic containing the data D, the result R, and the reduction predicate Red. It was shown that $D \Rightarrow R$ and $R \Rightarrow D$, so that $R = D$ (logical equivalence); one can also add that $D \Rightarrow \text{Red}$ and $\text{Red} \Rightarrow D$ in that special case. Furthermore, an interesting property whose importance was not yet realized was left unmentioned in my previous work, namely that, given any consistent logic containing the data D, one can always extend it by adding the predicates R and Red together with their negations to its field of propositions. One thus always obtains automatically another consistent logic (i.e., any further consistency condition resulting from this enlargement of the field of propositions is automatically satisfied because of the semidiagonal property of the interaction between the measured and the measuring systems) and the implications just mentioned hold in all these logics.

The questions raised by d'Espagnat have to do once again with an EPR situation.[16] In that connection, it should be realized that measuring, for instance, a spin component of a particle belonging to a two-particle system corresponds to a case where the eigenvalues of the measured observable are now degenerate. This property has to be taken into account in the discussion, which is, however, simplified by the kind of initial state used in EPR experiments. All that will be said later on comes from calculations corresponding to that case. Since these calculations are pretty easy, but somewhat lengthy, and since d'Espagnat does not criticize the technical aspects of my work, but its meaning, the results of "logical calculations" will be used here without further details.

5. ACTUAL FACTS

Perhaps the most important question in quantum mechanics has to do with the actualization, or if one prefers, the realization of a fact: How can it be that one specific fact occurs actually and is shown by a measuring apparatus, whereas quantum mechanics can only provide a density operator where all the possible outcomes of the experiment are kept on the same footing? I have given a tentative answer[10] bearing much similarity with the one proposed independently by Gell-Mann and Hartle.[9]

The point is that the probability given by Eq. (1) is not time-reversal invariant, so that there is a logical arrow of time. The separation of potential facts due to decoherence also has the same direction of time and it is known that decoherence and dissipation go together, so that the logical and the thermodynamic arrows of time fly parallel.

A consequence of this time asymmetry is the possibility to treat differently past and future. Given any time t, one can assume that all potential facts prior to t have been registered upon some records. The deterministic character of facts is essential to allow the existence of such records. One can then define many logics constructed as follows: each of them involves a *unique* set of potential facts at time t, including the records. This is enough to reconstruct any past preparation and measurement of a quantum system by logical means, i.e., without assuming actuality of the past facts, a logical reconstruction from records (memory) also makes them unique. Future, on the other hand, still consists only of potential facts with known probabilities in the corresponding histories envisioning it. The outcome is that the structure of quantum logic together with the properties of potential facts allow one to construct theoretically this kind of logical substructure without assuming the usual distinction between past and future as given from outside the theory. This is so similar to everyday experience that it is difficult to realize that what has just been described is an outcome of an abstract theory and not the intrusion of uncontrolled common sense in the theory.

At this point, one will of course identify actual facts with a unique particular subclass of potential present facts. (This identification looks very much like what a logician would call the transition from a formalized object language to a metalanguage rich enough to provide the formal one with a semantic.[17] So, we are quite near to what Tarski would have demanded before giving an unassailable definition of truth, although no formal discussion of that kind will be given here.) Whatever logicians would say, it is clear that the construction is deep enough to discuss efficiently the question of truth.

Actual observed facts can be used to assign a truth value to the propositions expressing or negating them. They will be held to be true.

The actualization of facts is the deepest problem in quantum mechanics. If I may venture here a personal opinion going outside physics, I shall say that it is perhaps intrinsically unsolvable and the mark that quantum mechanics is reaching some essential limit to knowledge: If theory provided a rule, a mechanism, a cause for the actualization of facts, physical reality would become essentially reduced or identified to its mathematical expression, the whole history of the universe being only the long reading of its initial state. By being intrinsically probabilistic, quantum mechanics is able to describe reality, perhaps as near as it can be without freezing reality into a Platonic, unmoving world of ideas. Of course, this is only an opinion and one can take it or leave it.

As far as the questions under discussion are concerned, one should clearly distinguish two approaches represented in this contest by d'Espagnat and the present author. The first approach relies upon educated common sense taking its roots in our nearly perfectly classical environment and culture to draw from it some philosophical views, for instance, realism. It can be applied to quantum mechanics in order to clear up these views or to question this theory, for instance, when considering hidden variables. The other approach has been made possible by the logical interpretation: it cannot criticize the foundations of quantum mechanics, because it is indissociably built in it, but it can directly question some simple or refined common sense concepts, since it can often find whether they can be proved, with what approximation, or disproved. I do not claim that one approach is better than the other, but they are so different that some problems arising from one of them can be meaningless in the other one.

In agreement with this general overview, the two points raised by d'Espagnat will be treated upon quite a different footing: his remarks about reliability and truth ask for more precision and rigor and they should be answered in full detail. His remarks concerning the validity of Bell's theorem are less important because, although Bell's theorem is a landmark from the standpoint of the first approach, it is not significant in the second one.

6. THE MEANING OF TRUTH

One can now state precisely what is a true property or a true proposition in quantum mechanics. First and foremost, a proposition expressing a potential fact, past or present, will be said to be true if it is realized by an actual fact. This is nothing but the famous recipe: "The rose is red" is true if the rose is red. However, this is certainly not enough in the case of quantum mechanics, because the facts only represent some properties of a macroscopic object and they never concern directly a microscopic system. This is why one is led in measurement theory to distinguish between the experimental *data*, which is a fact (as registered in some memory of a computer or just shown by a pointer on a dial), and the *result* of the experiment, which is the property of the measured system one wanted to get.

I propose to add to factual truths the following family of propositions: A proposition which does not simply state a fact will nevertheless be said to be *true* when it satisfies the following two conditions:

1. It can be added to the field of *any* consistent logic containing already all the facts to give a larger logic that is automatically consistent.
2. In this extended logic, the added proposition is logically equivalent to a fact.

I strongly suspect that condition 2 is unnecessary and that it is always a consequence of condition 1. These conditions are assumed for the following reasons: Assumption 1 means that the multiplicity of complementary logics is not any more a puzzling feature of the theory, since what is true is valid in all these logics. Every time the known facts (together with their formal negations) enter a logic, it is always possible to add to the corresponding field of propositions the propositions satisfying condition 1 together with their negation. Assumption 2 means in particular that the conditional probabilities that are necessary to prove the logical equivalences are the same in all these logics. Furthermore, their logical equivalence with the facts means that, from the standpoint of logic, they are mere redundancies and therefore necessarily true or false together with the facts. It means, for instance, that saying "The upper counter has registered" in a Stern–Gerlach experiment is strictly equivalent from a logical standpoint to saying "The z component of spin is $+1/2$."

From the standpoint of fundamental logic, assigning a truth value to a classical potential fact by looking at the actual facts at the present time (including records of past facts) is a consistent logical procedure in classical physics, although no formal proof of it exists to my knowledge. Once truth values have been given to present facts and their negation, determinism allows us to extend them to past facts. These truth values can be used in any consistent logic containing these asserted facts. When propositions satisfying conditions 1 and 2 are added to the logic which is used, whatever it may be, a truth value can be assigned to them because of their logical equivalence with factual propositions having already a truth value. These truth values are the same whatever the ambient logic, so that the corresponding notion of truth is *universal* without any arbitrariness. Since furthermore the truth tables are automatically correct, one can be sure that the elementary axioms of truth are valid in this construction. Of course, all this is valid only up to small known errors in probability: even truth is not perfect.

As opposed to a true proposition, a proposition will be said to be *reliable* when it is the logical consequence of a fact in *some* but not in all consistent logics.

An example will show what this means: Consider a spin 1/2 initially in the state $S_x = +1/2$; the z component of the spin is measured at a time t and is found to be $S_z = +1/2$. Let us introduce a logic L containing the predicates "$S_z = \pm 1/2$ at time t" and "$S_z = \pm 1/2$ at time t'," $0 < t' < t$. In this logic, the predicate $a =$ "$S_z = +1/2$ at time t'" is equivalent to the result "$S_z = +1/2$ at time t," itself equivalent to a fact, so that a is reliable. It is not true, however, because one can also use a logic L' including the result and its negation, but now the predicates "$S_x = \pm 1/2$ at time t'" referring to another spin component at the same time t'. This is the example considered by d'Espagnat. It turns out that, if L' is large enough to describe a measurement used to prepare the initial state, the predicate $b =$ "$S_x = +1/2$ at time t'" is also reliable as logically equivalent to the factual first measurement (Here I correct a more formal treatment given in ref. 2 because I am using a larger logic). The two logics L and L' are consistent, but no consistent logic includes both of them, so that one says that they are complementary. The two predicates (a, b) cannot be said to be true, since this would violate a basic axiom of truth, namely that when a is true and b is true, then "a and b" is also true. No proposition "a and b" exists in any consistent logic.

The predicate a is nevertheless reliable because when one chooses a logic where it makes sense, one will never meet a contradiction, because of the noncontradiction theorem. Once again, let us give examples: To the family of reliable propositions belong, for instance:

1. The proposition A saying that a particle starting from the origin in an initially isotropic state at time zero, which is measured to be near a point $\mathbf{x}$ at a time t, was on its way near a straight line going from the origin toward $\mathbf{x}$ at some time $t' < t$. See ref. 2.

2. The proposition B saying that, under the same conditions and at the same time t', the particle had its momentum directed near the direction of $\mathbf{x}$.

Of course, these two propositions belong to two complementary logics, which is why they are only reliable, showing how arbitrary it may be to choose one rather than the other.

Does it mean that reliable propositions have no interest? Not at all, because an experimentalist frequently uses them. What interests the experimentalist is not that proposition A is true, but that its contrary can be ruled out, namely that the particle followed a crooked path through the laboratory. One cannot say that proposition A is true, because that would provoke a rebuke by d'Espagnat; nor can one say that its negation $\bar{A}$, which is what really matters for the experimentalist, is false, because by definition, this is strictly the same as saying that A is true. The point is that $\bar{A}$ is untrue, meaning that its probability is zero in any logic including the facts where this probability has a meaning. This is useful and should not be thrown out with the bathwater, since it is used, for instance, when estimating experimental errors by Monte Carlo methods. Of course, no sensible experimentalist will resort to such subtleties, but it is important to be able to accept the estimated errors the experimentalist mentions and to believe the experimentalist's procedure from a more fundamental standpoint. This is why I still maintain the notion of reliability.

7. EPR EXPERIMENTS

The case of a proposition expressing essentially the reduction of the wave function, like the proposition Red mentioned previously, has quite a few interesting features: In a measurement of the first kind, it states simply that the measured observable has the value inferred from the data *at the end* of the measurement. It has been shown to be true in the case when one is measuring a nondegenerate observable A or when A is not correlated in the initial state with another observable commuting with it (like the spin and the momentum of a particle in many cases). In the case of an EPR experiment dealing with two strongly correlated particles, one must be very careful: It is still possible to state the value of a measured observable at the end of a measurement made on one particle and this is still a true proposition. However, what is ordinarily called wave packet reduction says more: it also states the correlated value of an observable belonging to the other particle. It will be shown that this is only reliable.

EPR experiments provide some good and rather clear examples of the usefulness of the present concepts and they will now be discussed in some detail, although the calculations necessary to substantiate the logical statements will not be given. Consider therefore two nonrelativistic spin-1/2 particles initially in a state of total spin zero. The spins of these particles are measured: the spin component of particle 1 (resp. 2) along a direction **a** (resp. **b**) is measured at time t_1 (resp. t_2). The proposition stating that the spin component of particle 1 along the direction **a** is equal to $+1/2$ at a time t will be denoted by $[1, \mathbf{a}, +, t]$, with obvious extensions to other cases. It will be assumed for definiteness that the measured data correspond to the results $[1, \mathbf{a}, +, t_1]$ and $[2, \mathbf{b}, +, t_2]$, the directions **a** and **b** not being parallel, which is the only interesting case (by the way, it is possible to say when **a** and **b** are near enough to be said parallel when the trace norm of the difference in the corresponding projectors is smaller than the error in the measuring projectors associated with the data).

The proof of reliability will proceed in several steps:

1. The propositions $[1, \mathbf{a}, +, t_1]$ and $[2, \mathbf{b}, +, t_2]$ expressing the results of the two measurements can be proved to be true with the above definition of truth.

Before going further, one may notice that it is always possible to choose $t_1 \leqslant t_2$ and the cases $t_1 < t_2$ and $t_1 = t_2$ have to be distinguished. The second case is not really interesting, because it amounts to the measurement of a nondegenerate couple of observables and it boils down to the trivial case already mentioned. Therefore, one will only consider the first case. The two measurements are assumed to be of the first kind.

2. Assuming that each measurement lasts a very short time Δt, the situation of particle 1 after the first measurement is described by the predicate $[1, \mathbf{a}, +, t_1 + \Delta t]$. It also turns out to be true.

3. It may be noticed that the proposition just mentioned in point 2 is not the complete expression of the reduction of the wave function. As used by EPR in their discussion, this reduction corresponds to the proposition "$[1, \mathbf{a}, +, t_1 + \Delta t]$ and $[2, \mathbf{a}, -, t_1 + \Delta t]$." To prove that it is reliable, one will have to show that it is equivalent to a fact in some logic but that it cannot be stated in another logic. This leads us to consider different logics:

4. Let us introduce a logic L containing the facts together with the two propositions $[1, \mathbf{a}, +, t_1 + \Delta t]$ and $[2, \mathbf{a}, -, t_1 + \Delta t]$. It can be shown to be consistent and the reduction proposition "$[1, \mathbf{a}, +, t_1 + \Delta t]$ and $[2, \mathbf{a}, -, t_1 + \Delta t]$" is logically equivalent in it to the factual data of the first measurement. This is the logic where one states what happens at the end of the first measurement and, taking into account the correlation in the initial state, one also makes a statement about the nonmeasured particle.

This is a logic frequently used when one believes in wave packet reduction and Einstein, Podolsky, and Rosen used it when they introduced their "elements of reality."[16]

5. One can extend this logic L by adding to it any number of propositions $[1, \mathbf{a}, +, t]$ for any time t such that $t \leqslant t_1$ or $t \geqslant t_1 + \Delta t$, which amounts to extending the result of the first measurement to what "occurred" to the measured particle before and after this measurement. One can also add to L any number of predicates $[2, \mathbf{a}, -, t]$ for any $t < t_2$. All of them turn out to be logically equivalent to the first factual measurement. One may notice that, curiously enough, there is more reliable knowledge about the particle which is not directly measured than about the directly measured particle: The interval of time during which the first measurement takes place is not excluded for the nonmeasured particle, whereas one must refrain from stating anything concerning a property of the measured particle during that interval of time. Of course, this is due to the interaction between the measured particle and the measuring apparatus so that nothing of that sort can be said because, during that time, the system to be described would have to contain the measuring device. This extended logic L is the one where one makes most of the first measurement.

6. There is another logic L' analogous to L making the best use of the second measurement. It contains predicates $[2, \mathbf{b}, +, t]$ for a time $t < t_2$ as well as $[1, \mathbf{b}, -, t]$ for $t < t_1$. It consists in extrapolating by retrodiction the results of the second measurement to the "properties" of the particle 1 first measured before its measurement! It is also consistent and obviously complementary to L. One may notice a strong analogy between these logics L and L' and the logics that were used previously to discuss a unique spin 1/2 from the standpoint of a measurement and of preparation. In the literature, these two logics are often associated with the points of view taken by two different observers performing one of the two measurements without exact knowledge of the other. Of course, this kind of approach through consciousness and/or information has nothing to do with the real questions at hand.

Comparing these results, one can see that these logical reconstructions of the past by different logics are somewhat arbitrary, as emphasized by d'Espagnat, and they are left to our freedom of choice, at least in the present case. The ordinary reduction postulate belongs to L, where it is logically equivalent to a fact and it cannot enter L'. Therefore, it is reliable, but not true. What is true besides the factual data reduces to the results of measurements $[1, \mathbf{a}, +, t_1]$ and $[2, \mathbf{b}, +, t_2]$ and the statement of a property of an actually measured particle just after measurement $[1, \mathbf{a}, +, t_1 + \Delta t]$ and $[2, \mathbf{b}, +, t_2 + \Delta t]$.

To conclude this discussion, one sees that the distinction between true propositions and reliable ones separates essentially what may be used to gain a knowledge of reality and what, although being free from contradictions, is however left to an arbitrary choice. This may be the choice of somebody one might call the observer or, better, the speaker. These results confirm what d'Espagnat said about the arbitrariness of reliable propositions. They also show that fortunately much can be said to be really true without the stain of arbitrariness.

There is also something quite interesting and rather unexpected: It is seen that d'Espagnat's criticism concerning reliability versus truth was much more destructive than one could expect: It does not affect the essential results of the logical interpretation, but it turns against the proposition stating the reduction of the wave function. It is only reliable and cannot be held to be true! The status of wave packet reduction in the logical approach where it appears as an unnecessary convenience does not turn this result into a catastrophe as it would do in the conventional form of the Copenhagen interpretation. In the same vein, when one remembers that Einstein, Podolsky, and Rosen had believed that this proposition provides the knowledge of an "element of reality," one also finds that EPR *elements of reality cannot be said to be true.*

8. CONCERNING SEPARABILITY

D'Espagnat's purported main concern in his article was the status of Bell's theorem about the nonseparability of quantum mechanics. His approach to this question was perhaps somewhat devious, since he started from the idea that "some people" might get the "impression" that this theorem was negated in the interpretations advocated by Griffiths and myself, although acknowledging that neither of us had written anything on the subject. In this he was quite insightful and he had anticipated here a relation I had not myself noticed. A more thorough analysis of this question may help to understand why this impression arose and what is the exact status of separability and Bell's theorem with respect to the logical interpretation.

The answer cannot be straightforward. One will have to distinguish carefully between two different notions of separability, namely *individual* and *statistical separability*, referring respectively to a specific individual system or to a statistical ensemble. Individual separability will be seen to have itself two meanings because it involves some properties of the system that may be either true or reliable. True (real) separability will be seen to hold in the logical interpretation while rejected in its arbitrary (reliable) version. On the other hand, statistical separability, which is the proper framework of Bell's theorem, will be seen to lie outside the domain and the reach of the present theory, which can only reject its assumptions. No wonder that such a shifty situation needing so many distinctive cases could only give rise to some sort of "impression" before the question could be clarified.

A rather clear statement of individual separability is given by d'Espagnat in his book.[18] A theory is said to be separable if it satisfies the following condition: When a physical system remains isolated during some time interval, the evolution of its properties during that interval cannot be influenced by operations carried on other systems. Although the exact meaning of some of the words used here might need more comments, I shall avoid this kind of discussion because there does not seem to be a significant disagreement here.

The previous discussion of the EPR experiment clearly shows what this separability condition means in the framework of the logical interpretation: If the "properties" mentioned in it are true, then separability is satisfied, i.e., no true proposition can correspond to a property of an isolated system that is influenced by an operation carried out upon another system. In fact, in a clear-cut sense of individual separability, quantum mechanics *is* separable. However, if the properties are also supposed to be reliable, then quantum mechanics remains nonseparable.

In practice, this means that the complete reduction of the wave packet in an EPR experiment is reliable but arbitrary and not true. If one insists that this assumed reduction nevertheless represents an influence of one subsystem upon the other, then one will say that quantum mechanics is not separable for an individual system. I reject this last point of view because I accept that anything containing arbitrariness should better be avoided, as advocated by d'Espagnat in his article.[1] From that standpoint, the individual nonseparability of quantum mechanics is only a matter of advancing arbitrary reliable propositions rather than sticking to what is true. It is just a way of speaking open to the choice of the speaker and one may refuse it without any factual or true consequence.

9. STATISTICAL SEPARABILITY

Let us now come to Bell's theorem. The statistical version of separability refers to statistical ensembles. I shall refer to an example rather than to the full generality of the theorem for clarity. A physical system S (say the initial system of two spin-1/2 particles in an EPR experiment) is first assumed to be "objectively" defined by some parameters λ. These parameters are often assimilated to hidden variables, although this is not necessary.[5] The theory is said to be separable if the probability for getting some result α in the measurement of an observable A concerning particle 1 has the property

$$\left|\frac{p(\alpha\mid\lambda,\beta)}{p(\alpha\mid\lambda)}-1\right|<\eta \tag{8}$$

where $p(\alpha\mid\lambda,\beta)$ is the conditional probability for finding the result α in the measurement when the source is in some objective state while another measurement of another observable performed upon particle 2 gives the result β, the quantity η being supposed to be small. Bell[6] took it to be zero, but d'Espagnat took it to be finite without mentioning that it might depend upon the system under study.

The conclusion of the theorem is that quantum mechanics is not statistically separable and there is no going against it.

Is this to suppose that the state of the system is not "objectively" defined? As far as the logical interpretation is only based upon facts without even needing them to be observed, it is certainly objective if one agrees that objectivity means taking facts and only facts into account. Therefore the notion of objectivity used in this theorem is not so obvious as it might look and one is just having a quarrel of language which comes from the irreductibility of the two approaches mentioned previously.

I propose to approach this question from my point of view as arising from a prejudice coming from unsufficiently criticized common sense. This means that one should understand why statistical separability holds in the classical limit, which is the mother of common sense, so as to see why the prejudice arose. (Once again, to avoid misunderstanding, it should be stressed that prejudiced common sense means here something that is commonly accepted by simple or educated common sense, but contrary to the "proof" of common sense obtained from the logical interpretation. It has no personal content.)

So, let us ask whether a macroscopic system exhibiting strong correlations (for instance, a shell breaking into two parts) can be objectively defined and whether condition (8) is satisfied in that case. The answer is twice yes, η being of the order of $(\hbar/L\Pi)^{1/2}$, L and Π expressing the allowed uncertainty upon the "hidden" variables which can be identified in that case: they are just the coordinates of position (including orientation) and momentum (including angular momentum) of the shell or, more precisely, parameters specifying a cell in phase space. The proof of this statement is a direct consequence of the theorems given in ref. 3.

This is obviously an approach which is just opposite to the spirit of Bell's theorem: using quantum mechanics, one proves that the *assumptions* of the theorem make sense in the classical case and one then finds that in that case separability holds. What was called "objectively defined" was the simultaneous assignment of position and momentum in a large cell in phase space, but this makes no sense for two small spins. Accordingly, the corresponding parameters λ have no analog in the case of small spins. So there is a deep difference between the macroscopic and the microscopic case and there is no way to transfer continuously from one to the other the concept of parameters, just as there is no reason to transfer the concept of statistical separability. Said otherwise, the parameter η entering Eq. (8) can be computed: it is small for a macroscopic system and of the order of unity for a microscopic one.

This important question is worth considering under other angles: even in the case of a macroscopic object, one cannot go to the limit of a pure quantum state, because statistical separability would not hold. Therefore the objective parameters really characterize a cell in phase space and not the points in it with their coordinates (Q, P). If there are N semiclassical quantum states in that cell, there are essentially N parameters taking the value 1 (for occupied state) or zero and the above value of η is of the order of $N^{-1/2n}$, n being the number of position coordinates. By giving the value of η, one gets the best that the logical interpretation can go toward its understanding of Bell's theorem. Except for that, it has nothing to say about it.

There is obviously a question arising as to the very different answers one gets for individual and statistical separability. My opinion is that people interested in these questions were perhaps too hasty when using the same name in both cases because they had the same kind of mental image in mind. It would be interesting if they could clarify this relation.

This is also not saying that Bell's theorem is of no interest; far from it. It only means that it belongs to a domain which lies outside the logical interpretation and conversely. When one starts from common sense and takes it seriously without too much worrying about its origin or its limits, one may be led to assume that anything, however small, is in a state that is "objectively" defined, for some meaning of that word in that case. This assumption leads to Bell's theorem and to a beautiful test of quantum mechanics. Since putting a theory to the test is always a very important contribution to physics, Bell's theorem is undoubtedly very important. Conversely, the logical interpretation must assume the veracity of some basic axioms of quantum mechanics to start with, so that it cannot even dream the assumptions of Bell's theorem except in the macroscopic case. In its proof of common sense, these assumptions appear to belong to the category of wrong prejudice which has to be discarded.

It also involves some internal criticism of quantum mechanics, but of a different kind: It finds out when the conventional version of the

Copenhagen interpretation must be subject to modifications and this can be put to experimental test, even if it turns out that these exceptions had been discovered previously by other means[11] and already confirmed experimentally (see, e.g., ref. 19), but this does not spoil the consistency of the approach.

10. CONCLUSIONS

To conclude:

1. I can only agree with d'Espagnat that one must be extremely careful in using the notion of truth when it comes to quantum mechanics and that the use Griffiths made of it was open to criticism, although that does not diminish in my opinion the great value of Griffiths' pioneering work.

2. The criticisms he directed against my own use of reliable propositions, showing that they depend upon an arbitrary choice left to the speaker or the observer, is correct. Although d'Espagnat was right in stressing that point, this was not a criticism against the actual content of my papers, since I had already emphasized it and this was precisely the reason why the word "reliable" instead of "true" was used.

3. One can define a notion of truth in quantum mechanics that exceeds mere factual truth. The true properties of a system do not depend upon a choice of logic and they can always enter any logic consistent with the facts. In that sense, they are universal and nonarbitrary. There are also reliable propositions which can never enter into a real logical contradiction; they are sometimes interesting or useful, but nevertheless arbitrary.

4. The propositions expressing the results of an experiment as derived from the factual data are always true.

5. The propositions expressing wave packet reduction are not always true. In particular, they are only reliable in the case of a first measurement occurring in an EPR experiment.

6. As far as the question of separability is concerned, it can be said that quantum mechanics is separable as far as the true properties of an individual system are concerned.

7. The assumptions of Bell's theorem, dealing with the possibility of statistical separability for a system which is supposed to have an "objective" state, have no clear meaning in the logical interpretation, so that this interpretation has nothing to say about it. It belongs to another conception of physics having no overlap with the present one. Accordingly, d'Espagnat's insistence concerning its importance in the present context was not to the point.

There will be no discussion of realism here, although it is at the root of these controversies, because this journal is not a convenient place for it.

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